C.U.SHAH UNIVERSITY
Summer Examination-2017

Subject Name : Complex analysis

Subject Code : 4SC05CAC1   **Branch: B.Sc (Mathematics)**

Semester : 5   **Date :   22/03/2017**   **Time : 02:30 To 05:30**   **Marks : 70**

Instructions:
    (1) Use of Programmable calculator & any other electronic instrument is prohibited.
    (2) Instructions written on main answer book are strictly to be obeyed.
    (3) Draw neat diagrams and figures (if necessary) at right places.
    (4) Assume suitable data if needed.

Q-1   **Attempt the following questions:**   **(14)**

    a) What is complex number? (01)
    b) If $z_1 = 4+5i$ and $z_2 = 5+4i$ then find $z_1 . z_2$. (01)
    c) Define : function of complex variables. (01)
    d) True/false : Every differentiable function is continuous. (01)
    e) What is imaginary part of complex number $2e^{i\frac{\pi}{4}}$? (01)
    f) What is analytic function? (01)
    g) Define : harmonic conjugate . (01)
    h) True/false : Every analytic function is differentiable. (01)
    i) What is real part of z^2 ? (01)
    j) Write the general Cauchy's integral formula. (01)
    k) Define : Entire function. (01)
    l) True/false : Every polynomial function is entire function. (01)
    m) Define: Bilinear transformation. (01)
    n) True/false : If the function is entire then it's integration on any closed curve is zero. (01)

Attempt any four questions from Q-2 to Q-8

Q-2   **Attempt all questions**   **(14)**
    (a) Define logz (02)
    (b) Show that $\lim\limits_{z \to 0} \frac{\bar{z}}{z}$ does not exist. (04)
    (c) Determine whether f '(z) exist and find its value when (08)
        (1) f(z) = $\frac{1}{z}$
        (2) f(z) = $x^2 + iy^2$

Q-3   **Attempt all questions**   **(14)**
    (a) Define: Exponential form of complex number. (02)
    (b) Find p such that f(z)=$r^2\cos2\theta + i\,r^2\sin p\theta$ is analytic. (04)
    (c) Prove that f(z)= e^z is entire function but f(z) = $e^{\bar{z}}$ is nowhere analytic. (08)

Q-4   **Attempt all questions**   **(14)**
   (a) Find f'(z) if f(z) = sinxcoshy + icosxsinhy .   **(02)**
   (b) Determine analytic function whose real part is e^xcosy   **(04)**

   (c) Show that if f(z)= u + iv is analytic then   **(08)**
          (1) u_x , u_y , v_x , v_y are continous.
          (2) $u_x = v_y$ and $u_y = - v_x$

Q-5   **Attempt all questions**   **(14)**
   (a) What is harmonic function?   **(02)**
   (b) What is polar form of complex number? State and prove Cauchy –Riemann   **(04)**
          condition in polar form
   (c) Prove that u = $\frac{x}{x^2 + y^2}$ is harmonic also find it's harmonic conjugate.   **(08)**

Q-6   **Attempt all questions**   **(14)**
   (a) if f(z) = u + iv is analytic then prove that   **(07)**
          $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4 |f'(z)|^2$
   (b) State and prove Cauchy's theorem.   **(07)**
Q-7   **Attempt all questions**   **(14)**
   (a) Find bilinear transformation w=f(z) which maps the points z = 1,i,-1 onto the   **(07)**
          points w= i ,0 ,-i respectively also write inverse transformation of it.
   (b) Evaluate :   **(07)**
          $\oint_C \frac{2z+1}{z(z-i)(z+i)}$ where C: $|z| = \frac{3}{2}$.

Q-8   **Attempt all questions**   **(14)**
   (a) State and prove Cauchy integral formula.   **(07)**
   (b) Evaluate :   **(05)**
          $\oint_C \frac{e^z}{(z-1)^2}$ where C: $|z| = 2$.
   (c) State ML-inequality .   **(02)**